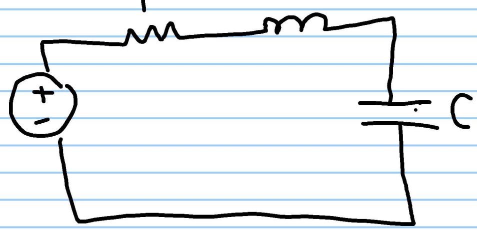
chapter 8

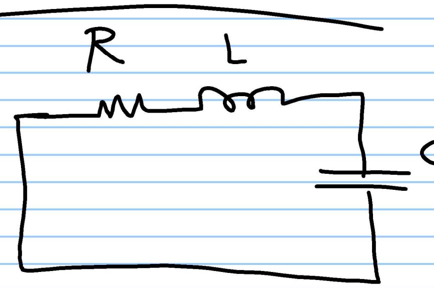
Natural & step Response of RLC circuits

2nd-order

series

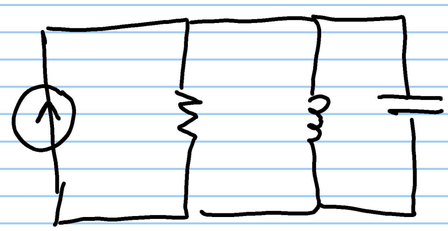


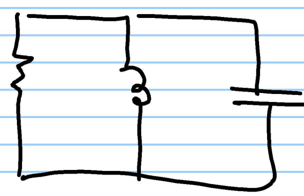
step



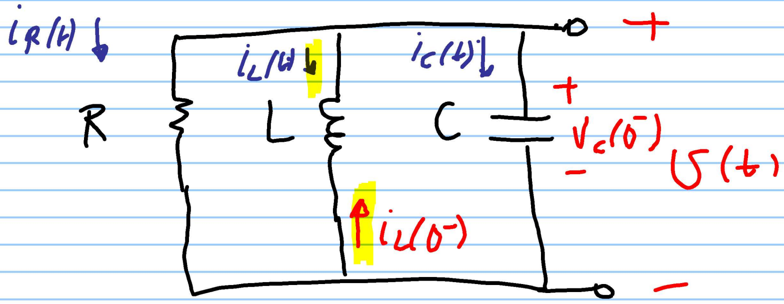
Naural

parallel





* Natural Response of parallel RLC circuits



$v_L = L \frac{di_L}{dt}$

$\frac{1}{L} v_L = \frac{d}{dt} i_L$

$\frac{1}{L}\int v + IC = i_L$

let $v_c(0^-) = 0$; $i_L(0^-) = 10\text{ A}$

KCL $\quad i_R(t) + i_L(t) + i_C(t)$

$$\frac{d}{dt}\left(\underbrace{\frac{v(t)}{R}} + \underbrace{\frac{1}{L}\int_0^t v(t)\,dt - i_L(0^-)} + \underbrace{C\frac{d}{dt}v(t)} = 0\right)$$

$$\frac{1}{R}\frac{d}{dt}v(t) + \frac{1}{L}v(t) + C\frac{d^2}{dt^2}v(t) = 0$$

$$\boxed{\frac{d^2}{dt^2}v(t) + \frac{1}{RC}\frac{d}{dt}v(t) + \frac{1}{LC}v(t) = 0}$$

2^{nd}-order homogeneous differential equation

$\therefore$ $\underline{v(t) = Ae^{st}}$ for $t>0$ $\qquad A_1e^{s_1t} + A_2e^{s_2t}$

$$As^2e^{st} + \frac{1}{RC}Ase^{st} + \frac{1}{LC}Ae^{st} = 0$$

$$Ae^{st}\left[s^2 + \frac{1}{RC}s + \frac{1}{LC}\right] = 0$$

$$\therefore\ s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \quad \text{characteristic equation}$$

$$s^2 + \underbrace{2\zeta\omega_n}_{2\zeta\omega_0}s + \underbrace{\omega_n^2}_{\omega_0^2}$$

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

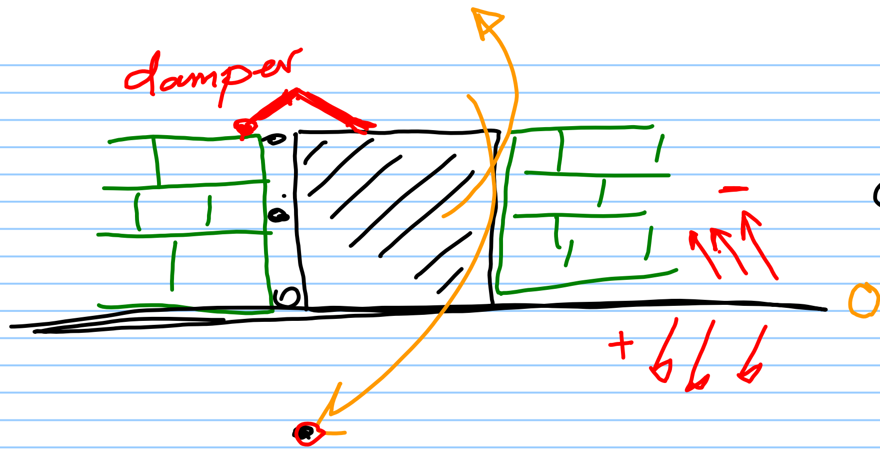
$$s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

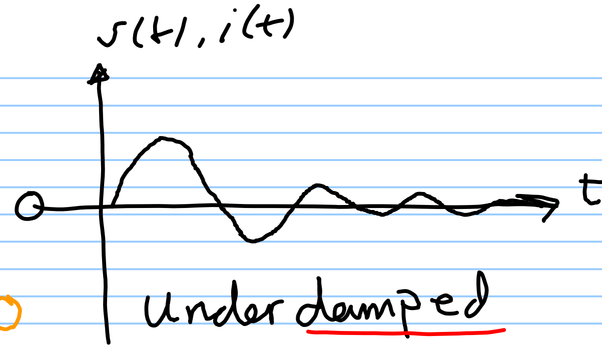
let $\omega_0 = \frac{1}{\sqrt{LC}}$ resonant frequency

& $\alpha = \frac{1}{2RC}$ damping coefficient

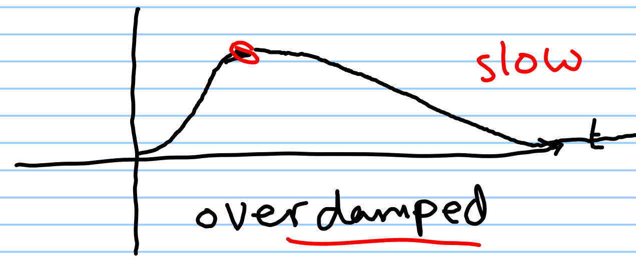
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$





Underdamped response



overdamped

critically damped

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

1) if $\alpha > \omega_0$, the solutions are real, unequal & the response is overdamped

2) if $\alpha < \omega_0$, the solutions are complex conjugates & the response is termed underdamped

3) if $\alpha = \omega_0$, the solutions are real and equal, & the response is termed critically damped.

$\boxed{1}$ overdamped case $\boxed{\alpha > \omega_0}$ $\alpha = \frac{1}{2RC}$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \quad -ve$$

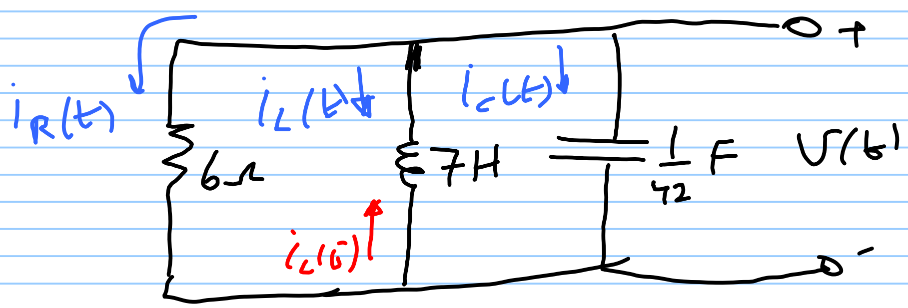
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \quad -ve$$

$\omega_0 = \frac{1}{\sqrt{LC}}$

real & unequal (real & distinct)

$$\therefore \boxed{v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}} \quad t > 0$$

EX| overdamped parallel RLC



if $i_L(0^-) = 10A$ & $V_C(0^-) = 0$. find $v(t)$ for $t > 0$

① $\alpha = \frac{1}{2RC} = 3.5$, $\omega_0 = \frac{1}{\sqrt{LC}} = 2.45$

$\alpha > \omega_0 \longrightarrow$ overdamped

② $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -1$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -6$

$$\therefore v(t) = A_1 e^{-t} + A_2 e^{-6t}, \quad t > 0$$

③ to find $A_1, A_2 \longrightarrow$ from the initial conditions

$v(0^+)$ & $\frac{d}{dt} v(0^+)$

$$\star \quad v(t) = A_1 e^{-t} + A_2 e^{-6t}$$

math

$$\left.\begin{aligned} &\rightarrow v(0^+) = A_1 + A_2 \\ &\rightarrow \frac{d}{dt} v(0^+) = -A_1 - 6A_2 \end{aligned}\right\}$$

$$s_1 A_1 + s_2 A_2$$

circuit

$v(0^+) \rightarrow$ The initial voltage on the capacitor

$d/dt\, v(0^+) \rightarrow$??

KCL $\quad i_c(t) + iR(t) + i_L(t) = 0$

$$\underbrace{C \frac{d}{dt} v(t)}_{i_c(0^+)} + \frac{v(t)}{R} + \frac{1}{L}\int_0^t v(t)\,dt \pm I_0 = 0$$

$t \rightarrow 0^+$

$$\underbrace{C \frac{d}{dt} v(0^+)}_{i_c(0^+)} + \frac{v(0^+)}{R} \mp I_0 = 0 \qquad v(0^+) = V_0$$

$$\therefore \frac{d}{dt} v(0^+) = \left(\mp I_0 - \frac{V_0}{R}\right)\frac{1}{C}$$

$$\boxed{\frac{d}{dt} v(0^+) = \frac{i_c(0^+)}{C} = \frac{\mp I_0 - \frac{V_0}{R}}{C}}$$

$$\rightarrow \frac{d}{dt} v(0^+) = \frac{-(-10)}{1/42} = 420$$

$$v(0^+) = A_1 + A_2$$

$$\frac{d}{dt} v(0^+) = -A_1 - 6A_2$$

$$A_1 + A_2 = 0 \quad \text{--- (1)}$$

$$-A_1 - 6A_2 = 420 \quad \text{--- (2)}$$

solving (1) & (2) $A_1 = 84$, $A_2 = -84$

$$\therefore v(t) = 84e^{-t} - 84e^{-6t} \ \text{V, for } t>0$$

$$= 84(e^{-t} - e^{-6t}) \ \text{V}$$

2 critical damping case

$$\alpha = \omega_0$$

$$s_{1,2} = -\alpha = -\frac{1}{2RC}$$

$s_1 = s_2$ real & equal

$$v(t) = A_1 t e^{-\alpha t} + A_2 e^{-\alpha t}$$

$\boxed{A_1 \ \& \ A_2} \Rightarrow v(0^+)$ & $d/dt \, v(0^+)$

$$\boxed{v(0^+) = A_2} = V_0$$

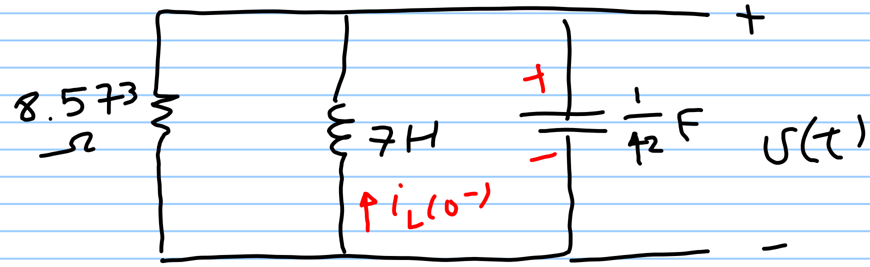
$$\frac{d}{dt} v(t) = (A_1 t)(-\alpha e^{-\alpha t}) + A_1 e^{-\alpha t} - \alpha A_2 e^{-\alpha t}$$

$$\boxed{\begin{aligned} d/dt \, v(0^+) &= A_1 - \alpha A_2 \\ &= \frac{i_c(0^+)}{c} \end{aligned}}$$

$$i_c(0^+) = -i_L(0^+) - \frac{v(0^+)}{R}$$

$$= -I_0 - \frac{V_0}{R}$$

EX1 critical damping Parallel RLC



$v_c(0^-)=0$, $i_L(0^-) = 10A$

$\alpha = \frac{1}{2RC} = \sqrt{6}$ $\qquad \omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{6}$

$s_1 = s_2 = -\sqrt{6}$ $\qquad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$\therefore v(t) = A_1 t e^{-\sqrt{6}t} + A_2 e^{-\sqrt{6}t}$ V, for $t>0$

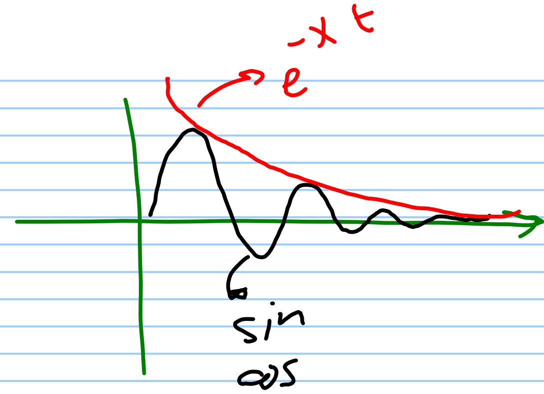
$v(0^+) = 0$ & $\frac{d}{dt}v(0^+) = 420$

$\boxed{v(0^+) = A_2 = 0}$

$\frac{d}{dt}v(0^+) = A_1 - \alpha A_2 = \boxed{A_1 = 420}$

$v(t) = 420 t e^{-\sqrt{6}t}$ V, for $t>0$

$\boxed{3}$ underdamped case



$$\alpha < \omega_0$$

$$\alpha^2 - \omega_0^2 < 0 \qquad (-ve)$$

$$\sqrt{\alpha^2 - \omega_0^2} = \sqrt{(-1)(\omega_0^2 - \alpha^2)}$$

$$= j\boxed{\sqrt{\omega_0^2 - \alpha^2}}$$

$$= j\,\omega_d$$

$\omega_d \equiv$ damped radian frequency

$$s_{1,2} = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$$

$$= -\alpha \pm j\,\omega_d \qquad \text{complex conjugate}$$

$$v(t) = A_1 e^{(-\alpha + j\omega_d)t} + A_2 e^{(-\alpha - j\omega_d)t}$$

$$e^{j\omega_d t} = \cos \omega_d t + j \sin \omega_d t$$

$$e^{-j\omega_d t} = \cos \omega_d t - j \sin \omega_d t$$

$$v(t) = e^{-\alpha t}\left[\underbrace{(A_1 + A_2)}_{B_1}\cos \omega_d t + \underbrace{j(A_1 - A_2)}_{B_2}\sin \omega_d t\right]$$

$$v(t) = e^{-\alpha t}\left[B_1 \cos \omega_d t + B_2 \sin \omega_d t\right]$$

NOTE

B_1 & B_2 are real #
Not complex

$$B_2 = j(A_1 - A_2) \quad (\text{real})$$

$$= j([x+jy] - [x-jy])$$

$$= j(2jy)$$

$$= -2y \quad \text{real } \#$$

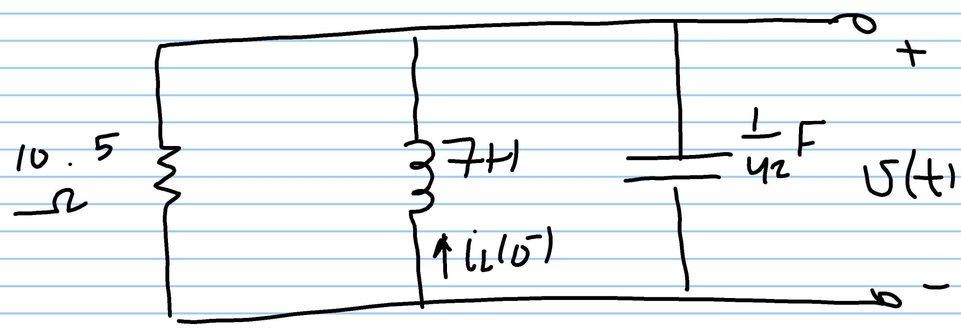
$$\therefore v(t) = e^{-\alpha t}\left[B_1 \cos \omega_d t + B_2 \sin \omega_d t\right]$$

B_1 & $B_2 \longrightarrow v(0^+)$ & $\frac{d}{dt} v(0^+)$

$$v(0^+) = B_1 = V_0$$

$$\frac{d}{dt} v(0^+) = \frac{i_c(0^+)}{C} = \frac{-I_0 - \frac{V_0}{R}}{C} = -\alpha B_1 + \omega_d B_2$$

EX | underdamped Parallel RLC



$v_c(0^-) = 0, \quad i_L(0^-) = 10A$

$$\alpha = \frac{1}{2RC} = 2, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{6} = 2.45$$

$\alpha < \omega_0$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} \therefore \sqrt{2}$$

$$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

$$= e^{-2t}(B_1 \cos \sqrt{2} t + B_2 \sin \sqrt{2} t)$$

$$v(0^+) = 0$$

$$\frac{d}{dt} v(0^+) = 420$$

$$v(0^+) = B_1 = 0$$

$$\therefore v(t) = e^{-2t} B_2 \sin \sqrt{2} t \quad V, \ t > 0$$

$$\frac{d}{dt} v(t) = (B_2 e^{-2t})(\sqrt{2} \cos \sqrt{2} t) + (\sin \sqrt{2} t)(-2 e^{-2t} B_2)$$

$$\frac{d}{dt} v(0^+) = 420 = \sqrt{2} B_2 \Rightarrow B_2 = \frac{420}{\sqrt{2}}$$

$$\therefore v(t) = \frac{420}{\sqrt{2}} e^{-2t} \sin \sqrt{2} t \ V, \ t > 0$$

$$s_{1,2} = -2 \pm j\sqrt{2}$$

Step Response of parallel RLC circuits

if $i_L(0^-) = 0$ & $v_c(0^-) = 0$

find $v(t)$ & $i_L(t)$ for $t > 0$

⇒ $i_L(t)$

KCL

$$I = i_R(t) + i_L(t) + i_c(t)$$

$$I = \frac{v(t)}{R} + i_L(t) + C\frac{d}{dt}v(t)$$

But $v_L(t) = v(t) = L\frac{d}{dt}i_L(t)$

$$I = \frac{L}{R}\frac{d}{dt}i_L(t) + i_L(t) + LC\frac{d^2}{dt^2}i_L(t)$$

$$\frac{d^2}{dt^2}i_L(t) + \frac{1}{RC}\frac{d}{dt}i_L(t) + \frac{1}{LC}i_L(t) = \frac{I}{LC}$$

$$i_L(t) = i_n(t) + i_f(t)$$

$i_n(t) \to$ natural response

$i_f(t) \to$ forced response

→ to find $i_f(t)$

let $i_L(t) = k$

$$0 + 0 + \frac{1}{LC}i_L(t) = \frac{I}{LC}$$

$\boxed{i_f(t) = I} = 24\,mA$

$\rightarrow$ to find $i_n(t)$

$$\frac{d^2}{dt^2} i_L(t) + \frac{1}{RC} \frac{d}{dt} i_L(t) + \frac{1}{LC} i_L(t) = 0$$

$$s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0$$

$$s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$-\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$s_1 = -20\,000$, $s_2 = -80\,000$

Real & distinct $\longrightarrow$ overdamped response

$$i_{L_n}(t) = A_1 e^{-20\,000t} + A_2 e^{-80\,000t}$$

$\therefore i_L(t) = i_{L_f}(t) + i_{L_n}(t)$

$$\boxed{i_L(t) = 24mA + A_1 e^{-20000t} + A_2 e^{-80000t}}$$

$\Rightarrow$ to find A_1 & A_2, we need $i_L(0^+)$ & $\frac{d}{dt} i_L(0^+)$

$$\boxed{i_L(0^+) = i_L(0^-) = 0}$$

$$v_C(0^+) = v_L(0^+) = L \frac{d}{dt} i_L(0^+)$$

$$v_C(0^+) = v_C(0^-) = 0 \qquad \therefore \boxed{\frac{d}{dt} i_L(0^+) = 0}$$

$$i_L(0^+) = 24mA + A_1 + A_2$$

$$0 = 24mA + A_1 + A_2 \qquad \boxed{A_1 + A_2 = -24mA} \text{ — } ①$$

$$\frac{d}{dt} i_L(t) = 0 - 20000 A_1 e^{-20000t} - 80000 A_2 e^{-80000t}$$

$$\frac{d}{dt} i_L(0^+) = \boxed{-20\,000 A_1 - 80\,000 A_2 = 0} \longrightarrow (2)$$

from (1) & (2) $\boxed{A_1 = -32\ mA\ ,\ A_2 = 8\ mA}$

$$\therefore\ i_L(t) = 24 - 32 e^{-20000t} + 8 e^{-80000t}\ mA\ ,\ \text{for } t > 0$$

$$V(t) = L \frac{d}{dt} i_L(t)$$

$$= 16 e^{-20000t} - 16 e^{-80000t}\ V,\ \text{for } t > 0$$

Example 8.10 Finding Step Response of a Parallel *RLC* Circuit with Initial Stored Energy

Energy is stored in the circuit in Example 8.8 (Fig. 8.12, with $R = 500\ \Omega$) at the instant the dc current source is applied. The initial current in the inductor is 29 mA, and the initial voltage across the capacitor is 50 V. Find (a) $i_L(0)$; (b) $di_L(0)/dt$; (c) $i_L(t)$ for $t \geq 0$; (d) $v(t)$ for $t \geq 0$.

a dc current source of 24 mA

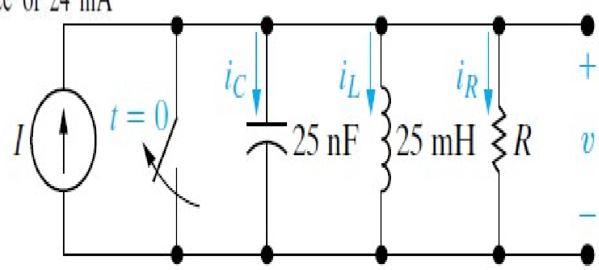


Figure 8.12 ▲ The circuit for Example 8.6.

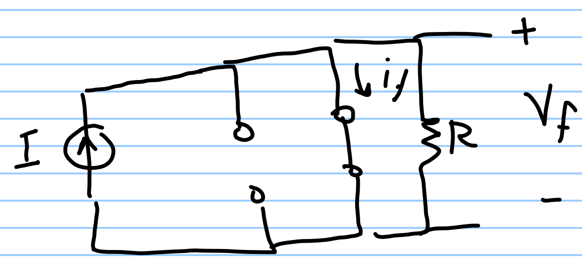
(a) $i_L(0) = i_L(0^+) = i_L(0^-) = 29\ mA$

(b) $v(t) = L \frac{d}{dt} i_L(t)$

$$\frac{d}{dt} i_L(t) = \frac{v(t)}{L} \Rightarrow \frac{d}{dt} i_L(0) = \frac{v(0)}{L} = \frac{50}{25m} = 2000\ A/s$$

(c) $i_L(t) = i_n(t) + i_f(t)$

$$i_f(t) = I = 24\ mA$$



α, ω

$\alpha = \frac{1}{2RC}$ $\qquad$ $\omega_0 = \frac{1}{\sqrt{LC}}$

$= 40000$ $\qquad$ $= 40000$

$\alpha = \omega_0 \Rightarrow$ critically damped.

$\therefore i_L(t) = i_f(t) + i_n(t)$

$= 24mA + A_1 t e^{-\alpha t} + A_2 e^{-\alpha t}$

$i_L(0) = 24mA + A_2 = 29mA$

$A_2 = 5mA$

$V_L = L \frac{d}{dt} i = V_C$

$\frac{d}{dt} i(0^+) = \frac{V_c(0^+)}{L}$

$\frac{d}{dt} i_L(0^+) = \frac{\cancel{V_c(0^-)}^{50}}{\cancel{L}_{25m}} = 2000$

$\frac{d}{dt} i_L(t) = \cancel{-A_1 t\alpha e^{-\alpha t}} + A_1 e^{-\alpha t} - \alpha A_2 e^{-\alpha t}$

$\frac{d}{dt} i_L(0) = A_1 - \alpha A_2 = 2000$

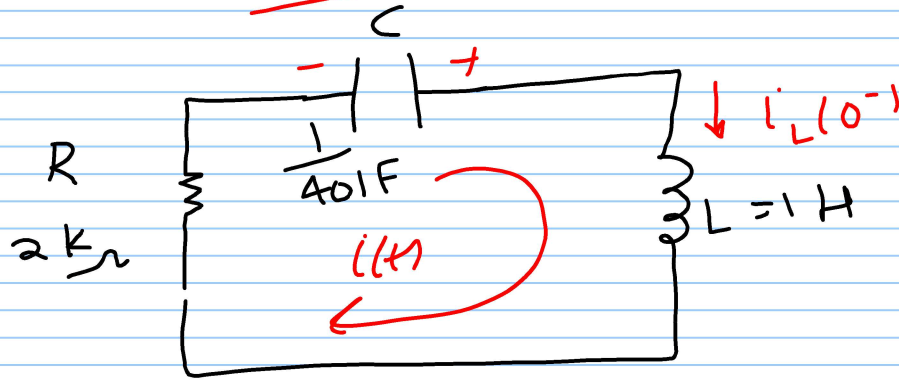
$A_1 - (40000)(5m) = 2000$

$\boxed{A_1 = 2200}$

$\therefore i_L(t) = 24mA + 2200 t e^{-40000t} + 5m e^{-40000t}$ A

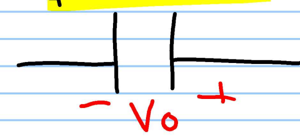
for $t > 0$

Natural Response of Series RLC circuits



$v_c(t)$

\+ −



if $V_c(0^-) = V_0$ & $i_L(0^-) = I_0$

find $i(t)$ for $t > 0$

$i_c = c \frac{d}{dt} v_c(t)$

$V_c = \frac{1}{c}\int i\,dt \underbrace{\mp v(0^-)}_{= V_0}$

KVL $\quad R\,i(t) + v_c(t) + L\frac{d}{dt} i(t) = 0$

$$\boxed{R\,i(t) + \frac{1}{c}\int_{0^-}^{t} i(t)\,dt - v_c(0^-) + L\frac{d}{dt} i(t) = 0} \quad \circledast$$

$$L\frac{d^2}{dt^2} i(t) + R\frac{d}{dt} i(t) + \frac{1}{c} i(t) = 0$$

$$\frac{d^2}{dt^2} i(t) + \frac{R}{L}\frac{d}{dt} i(t) + \frac{1}{LC} i(t) = 0$$

$$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$$

$\alpha = \frac{R}{2L}$ new

$$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$\omega_0 = \frac{1}{\sqrt{LC}}$ ✓

$$s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

let $v_C(0^-) = V_0 = 2V$

$i(0^-) = I_0 = 2mA$

$\alpha = \frac{R}{2L} = 1000$ $\qquad \alpha < \omega_0 \rightarrow$ underdamped case

$\omega_0 = \frac{1}{\sqrt{LC}} = 20025$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 20000$

$$i(t) = e^{-\alpha t}(\beta_1 \cos \omega_d t + \beta_2 \sin \omega_d t) \quad \text{for } t > 0$$

$$= e^{-1000t}(\beta_1 \cos 20000t + \beta_2 \sin 20000t) \quad \text{— } \boxed{A}$$

$\rightarrow$ to find B_1 & β_2, we need to have $i(0^+)$ & $\frac{d}{dt} i(0^+)$

$i(0^+) = i_L(0^-) = I_0 = 2mA$

$$\boxed{R\, i(t) + \frac{1}{C}\int_{0^-}^{t} i(t)\,dt - v_C(0^-) + L \frac{d}{dt} i(t) = 0} \quad \text{— } (*)$$

let $t = 0^+$

then, $R\, i(0^+) + 0 - V_0 + L \boxed{\frac{d}{dt} i(0^+)} = 0$

$$\frac{d}{dt} i(0^+) = \frac{V_0 - R\, i(0^+)}{L} = \frac{2 - 2000 \times 2m}{1} = -2$$

$\boxed{i(0^+) = 2\ mA}$ $\quad \boxed{\frac{d}{dt} i(0^+) = -2}$ $\Rightarrow$ equation A

$\rightarrow i(t) = e^{-1000t}(B_1 \cos 20000t + B_2 \sin 20000t)$

$\rightarrow i(0^+) = 1(B_1 \times 1 + B_2 \times 0)$

$\boxed{2m = B_1}$

$$\frac{d}{dt} i(t) = e^{-1000t}\left(-B_1 \times 20000 \sin 20000t + B_2 \times 20000 \cos 20000t\right) - 1000\, e^{-1000t}\left(B_1 \cos 20000t + B_2 \sin 20000t\right)$$

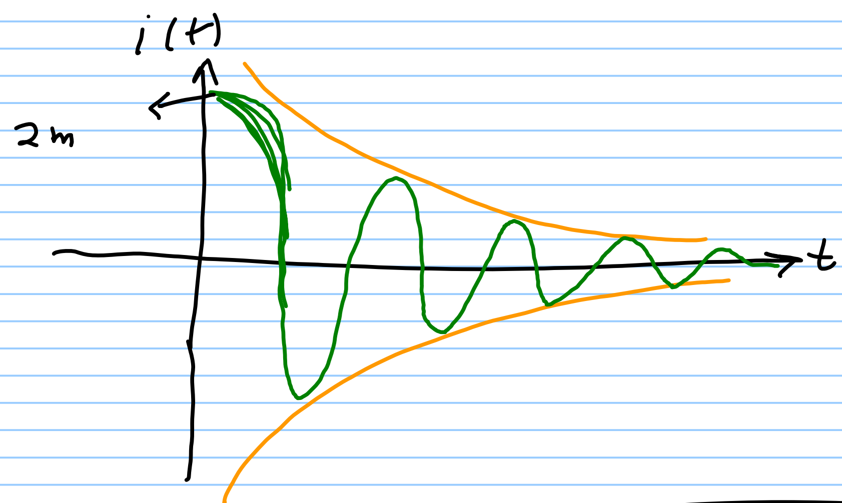
$B_2 \times 20\,000$

$$\therefore \frac{d}{dt} i(0^+) = \omega_d B_2 - \alpha B_1 = 20\,000 B_2 - 1000 B_1$$

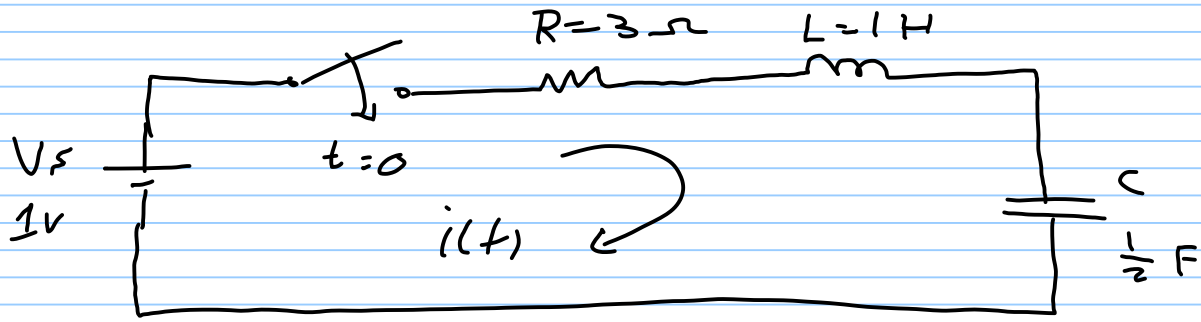
$$-2 = 20\,000 B_2 - 1000 \times 2 \times 10^{-3}$$

$$\boxed{B_2 = 0}$$

$$\therefore i(t) = 2e^{-1000t} \cos 20000t \text{ mA}, \text{ for } t>0$$



Step Response of Series RLC circuit



$V_c(0^-) = 0$ & $i_L(0^-) = 0$

find $i(t)$ for $\underline{t>0}$

$$\left(V_s = R\,i(t) + L\,\frac{d}{dt}i(t) + v_c(0^-) + \frac{1}{c}\int_{0^-}^{t} i(t)\,dt \right)\frac{d}{dt}$$

$$0 = R\,\frac{d}{dt}i(t) + L\,\frac{d^2}{dt^2}i(t) + \frac{1}{c}i(t)$$

$$i(t) = i_n(t)$$

$$0 = L s^2 + R s + \frac{1}{c}$$

$$s^2 + 3s + 2 = 0$$

$s_1 = -1$, $s_2 = -2$ Real & distinct

$\therefore$ overdamped case

$$\therefore i(t) = A_1 e^{-t} + A_2 e^{-2t} \quad \text{— (A)} \quad \text{for } t>0$$

$\boxed{OR}$

$$\alpha = \frac{R}{2L} = 1.5 \qquad \alpha > \omega_0$$

$$\omega_0 = \frac{1}{\sqrt{Lc}} = \sqrt{2}$$

to find A_1 & A_2, we need $i(0^+)$ & $\frac{d}{dt}i(0^+)$

$i(0^+) = i_L(0^-) = \text{zero} \longrightarrow$ sub in (A)

$$i(0^+) = \boxed{A_1 + A_2 = 0} \quad \text{— (1)}$$

$$v_s = R\,i(t) + L\frac{d}{dt}i(t) + V_0 + \frac{1}{c}\int_{0^-}^{t} i(t)\,dt$$

$t = 0^+$

$$V_s = R\,i(0^+) + L\frac{d}{dt}i(0^+) + V_0 + 0$$

$$\boxed{\frac{d}{dt}i(0^+) = \frac{V_s - V_0 - R\,I_0}{L}}$$

$$\frac{d}{dt}i(0^+) = \frac{V_s - 0 - R\times 0}{L} = \frac{V_s}{L} = \frac{1}{1} = 1$$

$$\boxed{\frac{d}{dt}i(0^+) = 1} \qquad \frac{d}{dt}\ (A)$$

$i(t) = A_1 e^{-t} + A_2 e^{-2t}$

$\frac{d}{dt}i(t) = -A_1 e^{-t} - 2A_2 e^{-2t}$ $\longrightarrow$ let $t = 0^+$

$\frac{d}{dt}i(0^+) = \boxed{-A_1 - 2A_2 = 1}$ — (2)

from (1) & (2) $\quad A_1 = 1, \quad A_2 = -1$

$\therefore\ i(t) = e^{-t} - e^{-2t}$ A, for $t > 0$

$\boxed{\text{find } v_c(t)}$ HW

$1 - 2e^{-t} + e^{-2t}$ V